SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2009

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 180 minutes.
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Express your answers in simplest exact form unless otherwise stated.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Start each **NEW** question in a separate answer booklet.

Total Marks - 120 Marks

- Attempt questions 1 - 10
- All questions are of equal value.

Examiner: *E. Choy*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Total marks – 120
Attempt Questions 1 - 10
All questions are of equal value

Answer each question/section in a SEPARATE writing booklet. Extra writing booklets are available.

SECTION A

Question 1 (12 marks)	Use a SEPARATE writing booklet	**Marks**

(a) Solve $\dfrac{2t}{5}+14=8$. **2**

(b) If $m_1=34$, $m_2=7$, $M=53$ and $g=9\cdot 8$, find correct to 4 significant figures the value of **1**

$$\left(\frac{m_1-m_2}{M+m_1+m_2}\right)g.$$

(c) The line $kx-2y=23$ passes through the point $(3,-1)$. **2**
Find the value of k.

(d) Simplify $\dfrac{x}{4}+\dfrac{3x-1}{3}$. **2**

(e) Factorise $3x^2+5x-12$. **2**

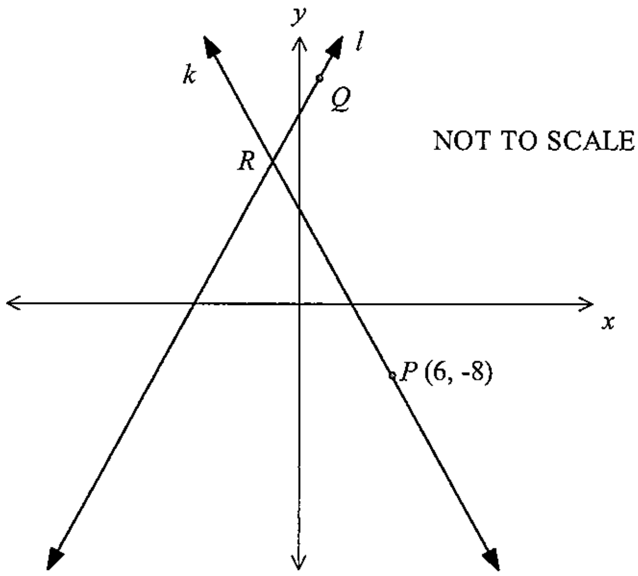
(f) Solve $7-4x>12$. **2**

(g) Write down the exact value of $\operatorname{cosec}\dfrac{\pi}{4}$. **1**

Question 2 (12 marks) **Marks**

(a) Solve $\tan x° = 1$ for $0° \le x° \le 360°$. **2**

(b) The diagram below shows the line $l: 2x - y + 8 = 0$ and the point $Q(2, 12)$ on it.
The line k has gradient -2 and passes through the point $P(6, -8)$.
The lines l and k intersect at R.



(i) Show that the equation of the line k is given by $2x + y - 4 = 0$. **1**

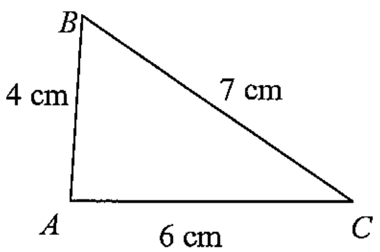
(ii) Show that the coordinates of R are $(-1, 6)$. **1**

(iii) Show that the distance QR is $3\sqrt{5}$. **1**

(iv) Find the perpendicular distance from P to the line l.
Leave your answer in simplified surd form. **2**

(v) Find the area of ΔPQR. **1**

(c) In the diagram below, ABC is a triangle in which $AB = 4$ cm, $BC = 7$ cm, and $CA = 6$cm.



(i) Use the Cosine Rule to show that $\cos C = \dfrac{23}{28}$. **1**

(ii) Write down the size of $\angle C$ correct to the nearest degree. **1**

(iii) Calculate the area of ΔABC.
Leave your answer correct to the nearest square centimetre. **2**

End of SECTION A

SECTION B

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate with respect to x

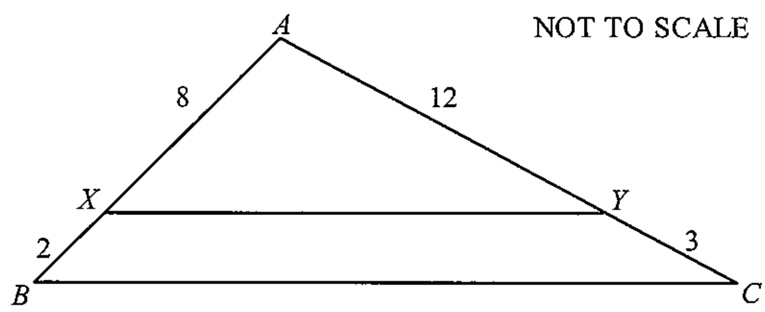
(i) $(3-x^2)^3$, **2**

(ii) $\log_e(x^2+3)$, **2**

(iii) $x\cos x$. **2**

(b) The graph of $y = f(x)$ passes through the point $(3, 5)$ and $f'(x) = 3-2x$. **2**
Find an expression for $f(x)$.

(c) In the diagram below, AB and AC are straight lines.
$AX = 8$, $BX = 2$, $AY = 12$ and $CY = 3$.



(i) Prove that $\Delta ABC \parallel\!| \Delta AXY$. **2**

(ii) Prove that XY is parallel to BC. **1**

(d) Find $\int \sqrt{x-6}\, dx$. **1**

Question 4 (12 marks) **Marks**

(a) Find the value of k if the quadratic equation $(x-3)(x+k)=k(x+2)$ has two equal roots. **2**

(b) After retiring from teaching Mathematics, Eric borrows \$130 000 to start a Shanghai Chinese restaurant. He is charged interest on the balance owing at the rate of $9 \cdot 75\%$ p.a. compounded monthly.
He agrees to repay the loan including the interest by making equal monthly instalments of \$$M$.

(i) How much does Eric owe at the end of the first month just before he pays his first instalment? **1**

(ii) Write an expression involving M for the total amount owed by Eric just after the first instalment is paid. **1**

(iii) Calculate the value of M (to the nearest cent) that which will repay the loan after 13 years. **3**

(iv) In how many months (to the nearest whole month) will the loan be repaid if Eric made instalments of \$1700 per month? **2**

(c) Sketch the parabola which

(i) has a focus of $(2,1)$ and directrix $x=4$. **1**

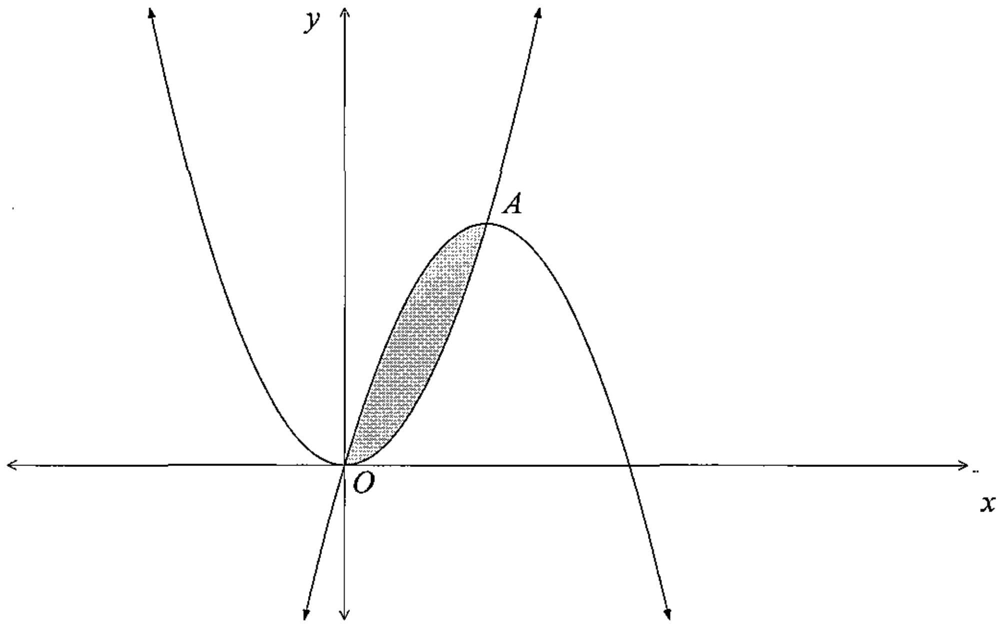
(ii) Find the equation of the parabola. **2**

End of SECTION B

SECTION C

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The diagram shows the curves $y = x^2$ and $y = 4x - x^2$, which intersect at the origin and at the point A.



(i) Show that the coordinates of the point A are $(2, 4)$ **2**

(ii) Hence find the area enclosed between the curves. **2**

(b) (i) Copy and complete the table of values for $y = \dfrac{1}{1+x^2}$.

Express your values in exact form. **2**

x	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
y					

(ii) Use Simpson's Rule with the five function values from part (i) to estimate **3**

$$\int_0^2 \frac{dx}{1+x^2}.$$

Give your answer correct to four decimal places.

(c) The sum of the first and third terms of a geometric series is 13. The sum of the second and fourth terms is $19\frac{1}{2}$. **3**

Find the first term and the common ratio.

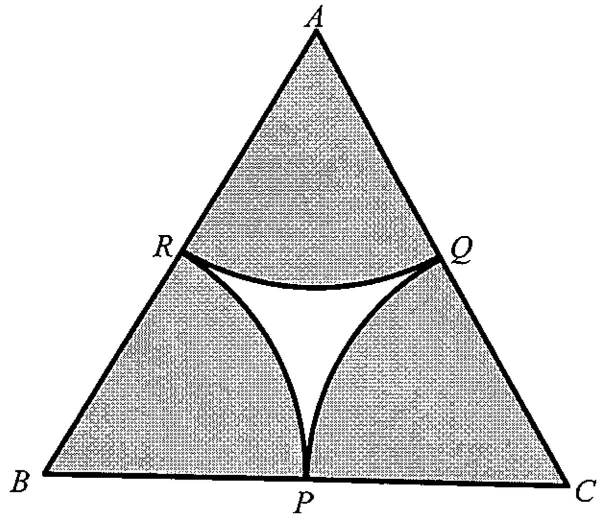
Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Prove $\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin\theta}$. **2**

(b) (i) Sketch the curves $y = \sin x$ and $y = \cos x$ for $0 \le x \le 2\pi$ on the same set of axes. **2**

(ii) Find the enclosed area bounded by the curves in part (i). **2**

(c) In the diagram below, triangle ABC is equilateral with a side length of 12 cm.
P, Q and R are the midpoints of BC, AC and AB respectively.
RP, PQ, and QR are arcs of circles centred at B, C and A respectively.



(i) Show that the area of triangle ABC is $36\sqrt{3}$ cm^2. **2**

(ii) Find the exact area of sector ARQ. **2**

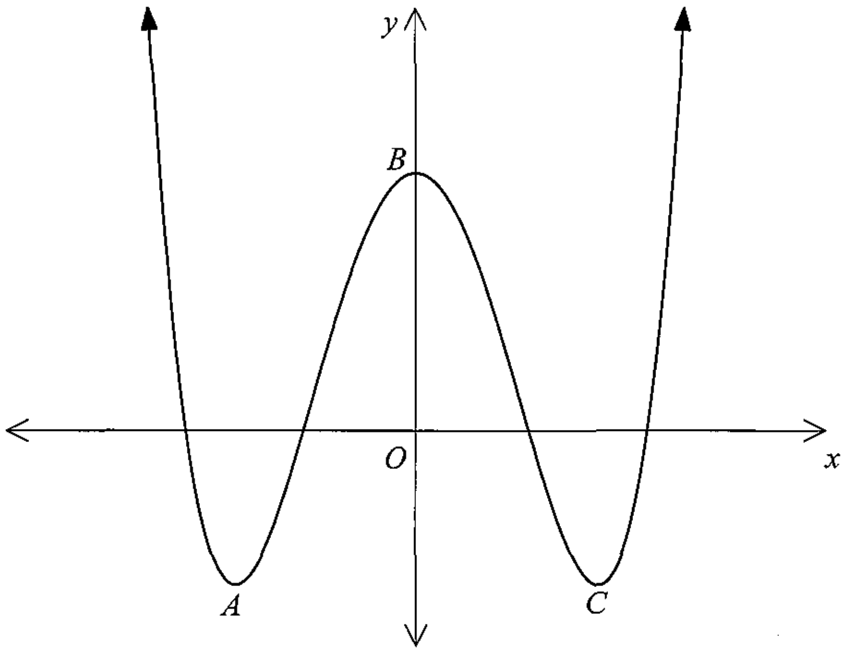
(iii) Hence find the area of the **unshaded part**, correct to three significant figures. **2**

End of SECTION C

SECTION D

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) The graph below is of the function $y = f(x)$ where $f(x) = x^4 - 8x^2 + 10$.
The points A and C are minimum turning points and B is the maximum turning point where the graph cuts the y-axis.



(i) Find the coordinates of B. **1**

(ii) Find $f'(x)$. **1**

(iii) Show that $f'(0) = f'(2) = f'(-2) = 0$. **2**

(iv) Hence find the coordinates of A and C. **2**

(b) Two bags contain respectively 5 red and 2 white balls, and 4 red and 1 white ball. One ball is drawn at random from each bag.

(i) Draw a probability tree diagram to show all the possibilities. **2**

(ii) Find the probability that the two balls drawn out are of different colours. **2**

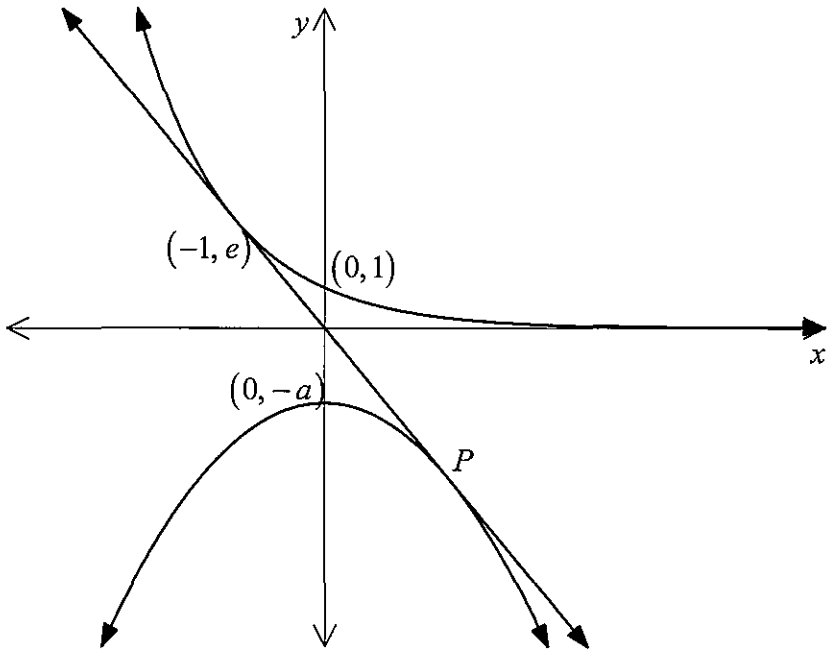
(c) A continuous curve $y = f(x)$ has the following properties for the closed interval $a \le x \le b$:

$$f(x) > 0,\ f'(x) > 0,\ f''(x) < 0.$$

Sketch a curve satisfying these conditions. **2**

Question 8 (12 marks) **Marks**

(a) The diagram below shows the graph of $y = e^{-x}$ and the parabola $y = -x^2 - a$.
The tangent to $y = e^{-x}$ through the point $(-1, e)$ is also the tangent to the parabola at P.



(i) Show that the equation of the tangent is $y = -ex$. **2**

(ii) Show that the value of x for which the tangent to $y = -x^2 - a$ has gradient $-e$ is $\frac{1}{2}e$. **2**

(iii) Find the coordinates of the point P, and hence find the value of a in exact form **3**

(b) The electrical charge Q retained by a capacitor t minutes after charging is given by $Q = Ce^{-kt}$, where C and k are constants.

The charge after 20 minutes is one half of the initial charge.

(i) Show that $k = \dfrac{1}{20}\ln 2$ **2**

(ii) How long will it be before one tenth of the original charge is retained? Answer to the nearest minute. **3**

End of SECTION D

SECTION E

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) A jet engine uses fuel at the rate of R litres per minute.
The rate of fuel use t minutes after the engine starts operating is given by

$$R = 15 + \frac{10}{1+t}.$$

(i) What is R when $t = 0$? **1**

(ii) What is R when $t = 9$? **1**

(iii) What value does R approach as t becomes very large? **1**

(iv) Draw a sketch of R as a function of t. **2**

(v) Calculate the total amount of fuel burned during the first 9 minutes. **2**
Give your answer correct to the nearest litre.

(b) The position x cm at time t seconds of a particle moving in a straight line is given by

$$x = 3t + e^{-3t}.$$

(i) Find the position of the particle when $t = 1$. **1**
Give your answer correct to 3 significant figures.

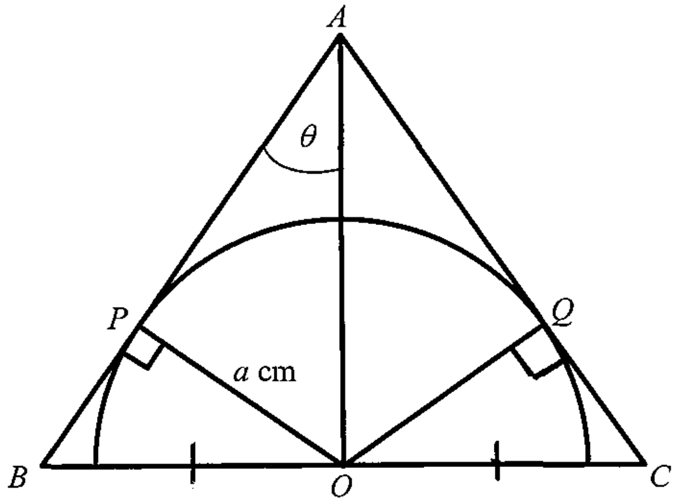
(ii) By finding an expression for the velocity of the particle, show that initially the particle is at rest. **2**

(iii) Find an expression for the acceleration of the particle. **1**

(iv) Find the limiting velocity of the particle as $t \to \infty$. **1**

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

ABC is a variable isosceles triangle with $AB = AC$.
The sides AB and AC touch a semicircle of radius a cm at P and Q.
O is the centre of the semicircle and BOC is a straight line.



Let S cm^2 be the area of ΔABC and $\angle BAO = \theta$.

It is given that $\sin 2\theta = 2\sin\theta\cos\theta$.

(a) Show that $S = \dfrac{2a^2}{\sin 2\theta}$, where $0 < \theta < \dfrac{\pi}{2}$. **3**

(b) Determine the range of values of θ for which S is
 (i) Increasing, **2**

 (ii) Decreasing. **2**

(c) Sketch the curve of S against θ for $0 < \theta < \dfrac{\pi}{2}$. **2**

(d) If $2a < OA < 3a$, find the greatest value of S. **3**

End of paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ \ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x,\ \ x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax},\ \ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,\ \ a \neq 0$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

Section A

Question 1.

a) Solve $\frac{2t}{5} + 14 = 8$

$\frac{2t}{5} = -6$ 1

$2t = -30$

$\boxed{t = -15}$ 1

b) $\left(\frac{34-7}{53+34+7}\right) \times 9.8$

$= 2.814893617$

$\boxed{= 2.815 \quad \text{4 sig fig}}$ 1

c) $3k - 2 \times -1 = 23$

$3k + 2 = 23$ 1

$3k = 21$

$\boxed{k = 7}$ 1

d) $\frac{x}{4} + \frac{3x-1}{3}$

$= \frac{3x + 4(3x-1)}{12}$ 1

$= \frac{3x + 12x - 4}{12}$

$\boxed{= \frac{15x - 4}{12}}$ 1

e) Factorise -36

$3x^2 + 5x - 12$

$= \frac{(3x+9)(3x-4)}{3}$ 1

$= \frac{\cancel{3}(x+9)(3x-4)}{\cancel{3}}$

$= \boxed{(x+9)(3x-4)}$ 1

f) $7 - 4x > 12$

$-4x > 5$ 1

$\boxed{x < -5/4}$ 1

g) $\operatorname{cosec} \pi/4$

$= \frac{1}{\sin \pi/4}$

$= \frac{1}{1/\sqrt{2}}$

$= \boxed{\sqrt{2}}$ 1

Question 2

a) $\tan x° = 1 \quad 0° \leq x \leq 360°$

S | A
T | C

$x = 45°, 225°$ (2)

b) i) $m = -2 \quad (6, -8)$

$y + 8 = -2(x - 6)$
$y + 8 = -2x + 12$

$\boxed{2x + y - 4 = 0}$ (1)

ii) $k: 2x + y - 4 = 0$
$l: 2x - y + 8 = 0$

$k + l \quad 4x + 4 = 0$
$4x = -4$

$\boxed{x = -1}$ (2)

sub into k, $-2 + y - 4 = 0$
$\boxed{y = 6}$

$\therefore \boxed{R = (-1, 6)}$ (1)

iii) ~~D~~ $Q(2, 12) \quad R(-1, 6)$

$D = \sqrt{(2+1)^2 + (12-6)^2}$

$= \sqrt{3^2 + 6^2}$

$= \sqrt{45} = 3\sqrt{5}$ (1)

iv) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

line: $2x - y + 8 = 0$
point: $(6, -8)$

$d = \dfrac{|2 \times 6 + (-1) \times (-8) + 8|}{\sqrt{2^2 + (-1)^2}}$

$= \dfrac{|12 + 8 + 8|}{\sqrt{5}}$

$= \dfrac{28}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \boxed{\dfrac{28\sqrt{5}}{5}}$ (2)

v) $A = \frac{1}{2} \times 3\sqrt{5} \times \dfrac{28\sqrt{5}}{5}$

$= 42\ u^2$ (1)

c) i $\cos C = \dfrac{6^2 + 7^2 - 4^2}{2 \times 6 \times 7}$

$= \dfrac{69}{84}$

$= \boxed{\dfrac{23}{28}}$ (1)

ii) $C = 34.77194403$
$= \boxed{35°}$ nearest degree. (1)

iii) $A = \frac{1}{2} ab \sin C$
$= \frac{1}{2} \times 7 \times 6 \times \sin 35$
$= 12.04510516$
$= \boxed{12 cm^2}$ (2)

3. HSC Trial 2 unit maths 2009

(a) (i) $\frac{d}{dx}(3-x^2)^3 = 3(3-x^2)^2 \times -2x$

$= -6x(3-x^2)^2$ 2

(ii) $\frac{d}{dx}(\log_e(x^2+3)) = \frac{2x}{x^2+3}$ 2

(iii) $\frac{d}{dx}(x\cos x) = x \times -\sin x + \cos x \times 1$

$= -x\sin x + \cos x$ 2

(b) $f'(x) = 3-2x$

$f(x) = \int(3-2x)\,dx$

$= 3x - x^2 + C$

data $(3,5)$

$5 = 9 - 9 + C$

$C = 5$

$f(x) = 3x - x^2 + 5$ 2

(c) (i) In $\triangle ABC \parallel\!\mid \triangle AXY$

$X\hat{A}Y = B\hat{A}C$ common angle

$\frac{AX}{AB} = \frac{8}{10} = \frac{4}{5}$

$\frac{AY}{AC} = \frac{12}{15} = \frac{4}{5}$

Common angle, sides in same ratio test. 2

$\triangle ABC \parallel\!\mid \triangle AXY$

(ii) Because $A\hat{X}Y = A\hat{B}C$ angles in corresp position

$\therefore XY \parallel BC$ 1

3 (d) $\int (x-6)^{\frac{1}{2}}\,dx$

$= \dfrac{(x-6)^{1\frac{1}{2}}}{1\frac{1}{2} \times 1} + c$

$= \frac{2}{3}(x-6)\sqrt{x-6} + c$ (1)

T12.

4 (a) $(x-3)(x+k) = k(x+2)$

$x^2 + xk - 3x - 3k = kx + 2k$

$x^2 + \cancel{xk} - \cancel{kx} - 3x - 3k - 2k = 0$

$x^2 - 3x - 5k = 0$

equal roots $\Rightarrow \Delta = b^2 - 4ac = 0$

$a = 1$

$b = -3$

$c = -5k$

$9 - 4 \times 1 \times -5k = 0$

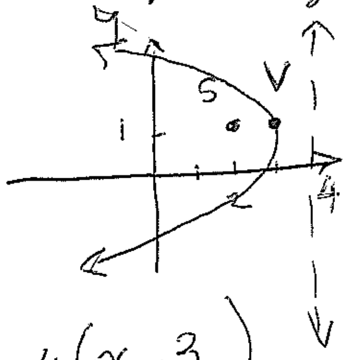
$9 + 20k = 0$

$20k = -9$

$k = -\frac{9}{20}$ (2)

(c) (i) $S(2,1)$

$x = 4$



(1)

$(y-k)^2 = -4a(x-h)$

(ii) $(y-1)^2 = -4(x-3)$ (2)

$a = 1$

$V = (3,1)$

4 (b) borrows \$130,000

9.75% p.a compounded monthly $\Rightarrow \frac{9.75}{12}\% = 0.008125$

equal monthly instalments \$m.

(i) $\$A_1 = 130,000 + 130,000 \times 0.008125$

$= 130,000(1 + 0.008125)$

$= 130,000(1.008125) = \$131056.25$ 1

(ii) $\$130000(1.008125) - m$. 1

(iii) 13 years = 156 months

$\$A_2 = 130000(1.008125)^2 - m(1 + 0.008125)$

$\$A_{156} = 130000(1.008125)^{156} - m(1 + 1.008125 + \ldots 1.008125^{155})$

$\$A_{156} = 0$

$$m = \frac{130000(1.008125)^{156}}{1 + 1.008125 + \ldots + (1.008125)^{155}}$$

denom: $S_n = \frac{rl - a}{r - 1} = \frac{1.008125 \times 1.008125^{155} - 1}{1.008125 - 1}$

$= \frac{1.008125^{156} - 1}{0.008125} = 311,8543626$

$\frac{a(r^n - 1)}{r - 1}$

$= \frac{1(1.008125^{156} - 1)}{1.008125 - 1}$

$m = \$1473.11$ 3

$\$A_n$

(iv) $130000(1.008125)^{n} - 1700(1 + 1.008125 + \cdots + \cdots 1.008125^{n-1})$.

let $\$A_n = 0$

$$1700(1 + 1.008125 + \cdots + 1.008125^{n-1}) = 130000(1.008125)$$

sum $1 + 1.008125 + \cdots + 1.008125^{n-1}$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1.(1.008125^n - 1)}{1.008125 - 1} = \frac{1.008125^n - 1}{.008125}$$

$$\frac{1700(1.008125^n - 1)}{0.008125} = 130000(1.008125)^n$$

$$1700(1.008125^n - 1) = 1056.25(1.008125)^n$$

$$1700(1.008125^n)^n - 1700 = 1056.25(1.008125)^n$$

$$643.75(1.008125)^n = 1700$$

$$1.008125^n = 2.64077669$$

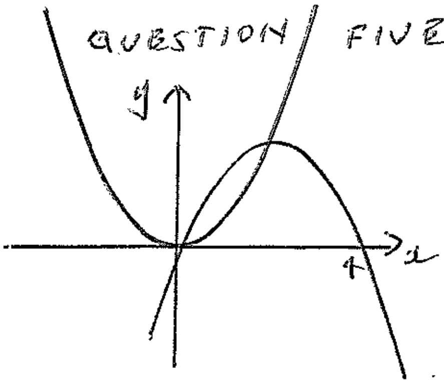
$$n \log 1.008125 = \log 2.64077669$$

$$n = 120 \text{ months}$$

2

/12

QUESTION FIVE



$y = x^2$

$y = 4x - x^2$

$4x - x^2 = x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0, 2$

$y = 0, 4$

Point is $(2, 4)$ (2)

$$A = \int_0^2 (4x - x^2)\,dx - \int_0^2 x^2\,dx$$

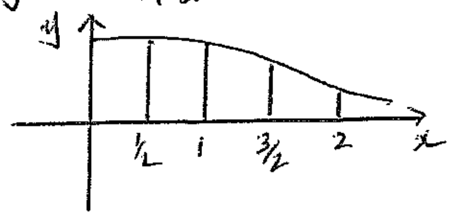
$$= \int_0^2 (4x - 2x^2)\,dx$$

$$= \left[2x^2 - \frac{2}{3}x^3\right]_0^2$$

$$= 8 - \frac{16}{3}$$

$= \frac{8}{3}$ square units (2)

$$f(x) = \frac{1}{1+x^2}$$



x	0	1/2	1	3/2	2	
y	1	4/5	1/2	4/13	1/5	

(2)

$$A \approx \frac{1}{6}(1-0)\left(1 + 4\times\frac{4}{5} + \frac{1}{2}\right) + \frac{1}{6}(2-1)\left(\frac{1}{2} + 4\times\frac{4}{13} + \frac{1}{5}\right)$$

$$= \frac{1}{6}\left(4\tfrac{7}{10}\right) + \frac{1}{6}\left(1\tfrac{121}{130}\right)$$

$$= \frac{47}{60} + \frac{251}{780}$$

$$= 1\tfrac{41}{390}$$

$= 1.1051$ (4dp)

OR $h = \dfrac{2-0}{4} = \dfrac{1}{2}$

$$A \doteqdot \frac{h}{3}\left[\left(1 + \frac{1}{5}\right) + \left(2\times\frac{1}{2}\right) + 4\left(\frac{4}{5} + \frac{4}{13}\right)\right]$$

$$= \frac{1}{6}\left(\frac{431}{65}\right)$$

$$= 1\tfrac{41}{390}$$

$= 1.1051$ (4dp)

NOTE: EXACT ANSWER IS $1.107148\ldots\ldots$

OR USING DECIMALS

$$A \doteqdot \frac{1}{6}\left[1 + 3.2 + 0.5\right] + \frac{1}{6}\left[.5 + 1.23077 + 0.2\right]$$

$$= \frac{1}{6}\left[6.63077\right]$$

$= 1.1051$

AND

$$A \doteqdot \frac{1}{6}\left[1 + .2 + 2\times .5 + 4(.8 + .30769)\right]$$

$$= \frac{1}{6}\left[6.63076\right]$$

$= 1.1051$ (3)

c) $a(1+r^2) = 13$ — ①

$ar(1+r^2) = \dfrac{39}{2}$ — ②

From ①

$$a = \frac{13}{1+r^2}$$

In ② $\dfrac{13}{1+r^2}\cdot r(1+r^2) = \dfrac{39}{2}$

$13r = \dfrac{39}{2}$

$r = \dfrac{3}{2}$

$a = \dfrac{13}{1+9/4} = 4$ (3)

Series is $4 + 6 + 9 + 13\tfrac{1}{2}$

$T_1 + T_3 = 13$, $T_2 + T_4 = 19\tfrac{1}{2}$

6) a) $\text{LHS} = \dfrac{\sin\theta}{1-\cos\theta} \times \dfrac{1+\cos\theta}{1+\cos\theta}$

$= \dfrac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$ $\quad \sin^2\theta + \cos^2\theta = 1.$

$= \dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$

$= \dfrac{1+\cos\theta}{\sin\theta}$

$= \text{RHS}$

OR

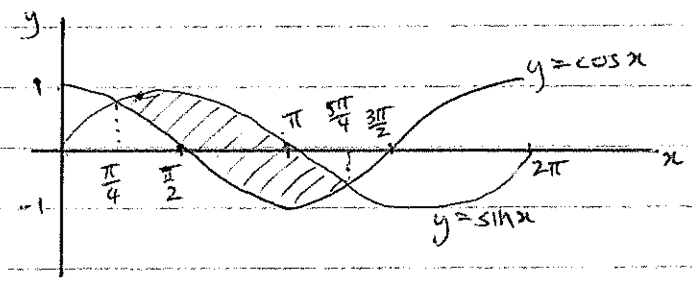
$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta = (1-\cos\theta)(1+\cos\theta)$

$\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin\theta}$

b) i)



ii) $\text{Area} = \displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx$

$= \Big[-\cos x - \sin x\Big]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$

$= -\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4} - \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$

$= -\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right) - \left(-\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\right)$

$= \dfrac{4}{\sqrt{2}}$

$= 2\sqrt{2} \text{ units}^2$

c) i) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2}(12)(12)\sin 60^\circ$

$= 72\left(\frac{\sqrt{3}}{2}\right)$

$= 36\sqrt{3}$ cm^2

ii) Area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(6)^2 \cdot \frac{\pi}{3}$

$= 6\pi$ cm^2

iii) Area $= 36\sqrt{3} - 3(6\pi)$

$= 5.81$ cm^2 (to 3 sig. figures)

SECTION D

QUESTION 7.

(a) $f(0) = 10.$

$B(0, 10)$ 1

(ii) $f(x) = x^4 - 8x^2 + 10$

$f'(x) = 4x^3 - 16x$ 1

(iii) $f'(0) = 0$

$f'(2) = 4 \times 8 - 16 \times 2$

$= 0$

$f'(-2) = -4 \times 8 + 16 \times 2.$

$= 0.$ 2

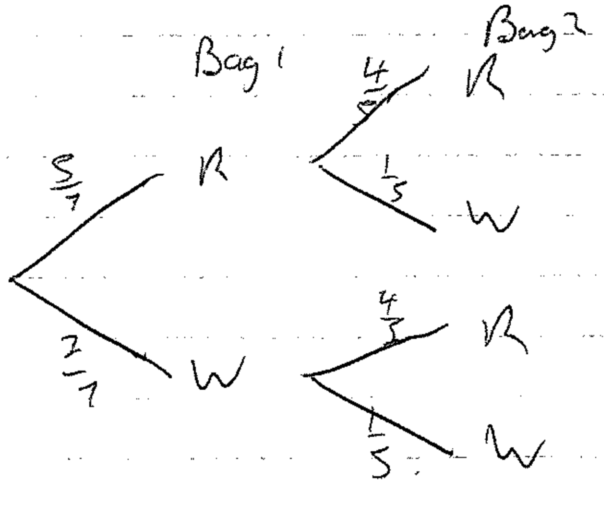
(iv) $f(2) = 16 - 8 \times 4 + 10$

$= -6$

$f(-2) = -6.$

$A(-2, -6)$ $C(2, -6)$ 2

(b) (i)



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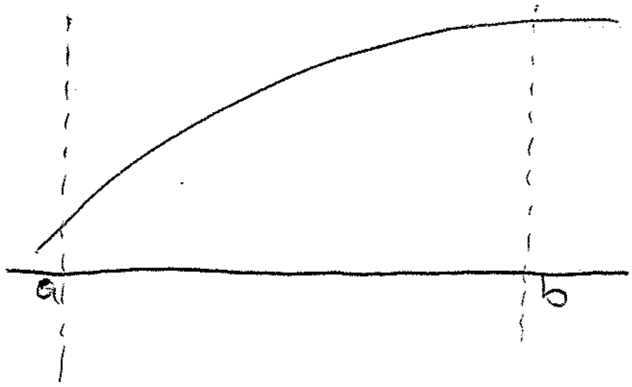
(ii) $P(RW) + P(WR) = \frac{5}{7 \times 5} + \frac{2 \times 4}{7 \times 5}$ 2

$= \frac{13}{35}$

(c)



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QUESTION 8

(a) (i) $\frac{dy}{dx} = -e^{-x}$.

at $x = -1$
$m = -e$.

$y - e = -e(x+1)$.

$y - e = -ex - e$.
$y = -ex$. 2

(ii) $\frac{dy}{dx} = -2x$

$-2x = -e$

$x = \frac{e}{2}$. 2

(iii) $y = -e \times \frac{e}{2}$. from tangent.

$= -\frac{e^2}{2}$. $P\left(\frac{e}{2}, -\frac{e^2}{2}\right)$. 1

$-\frac{e^2}{2} = -\left(\frac{e}{2}\right)^2 - a$.

$-\frac{e^2}{2} = -\frac{e^2}{4} - a$

$a = \frac{e^2}{2} - \frac{e^2}{4}$

$a = \frac{e^2}{4}$. 2.

(b) at $t=0$ $Q = Q_0$.

$Q_0 = Ce^0$.

i.e. $C = Q_0$.

$Q = Q_0 e^{-kt}$.

at $t=20$ $\frac{Q_0}{2}$

$\frac{Q_0}{2} = Q_0 e^{-20k}$.

$e^{-20k} = \frac{1}{2}$

$-20k = \ln \frac{1}{2}$.

$k = \frac{1}{20} \ln 2$. 2

(ii). $\frac{Q_0}{10} = Q_0 e^{-kt}$.

$\frac{1}{10} = e^{-kt}$

$-kt = \ln \frac{1}{10}$

$t = \frac{20 \ln 10}{\ln 2}$

$= 66$ mins 3

Question 9

(a) $R = 15 + \dfrac{10}{1+t}$

(i) $R = 15 + \dfrac{10}{1} = 25$

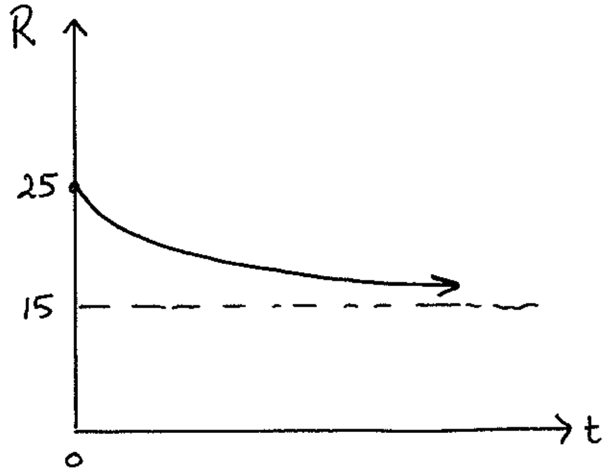
(ii) $R = 15 + \dfrac{10}{1+9} = 16$

(iii) As $t \to \infty$

$R \to 15$

since $\dfrac{10}{1+t} \to 0$

(iv)



(v) $\displaystyle\int_0^9 \left(15 + \frac{10}{1+t}\right) dt$

$= \Big[15t + 10\log_e(1+t)\Big]_0^9$

$\doteqdot 158\text{ L}$

(b) $x = 3t + e^{-3t}$

(i) When $t = 1$, $x = 3 + e^{-3}$

ie $x \doteqdot 3.05$

(ii) $V = \dfrac{dx}{dt} = 3 - 3e^{-3t}$

When $t = 0$, $V = 3 - 3e^{0}$

$= 3 - 3(1)$

$V = 0$

$\therefore$ initially at rest.

(iii) $\ddot{x} = \dfrac{dv}{dt} = 9e^{-3t}$

(iv) $\displaystyle\lim_{t \to \infty}\left(3 - 3e^{-3t}\right)$

$= \displaystyle\lim_{t \to \infty}\left(3 - \frac{3}{e^{3t}}\right)$

$= 3$

$\left(\text{Since } \dfrac{3}{e^{3t}} \to 0\right)$

Q10. (a) $S = \frac{BC \times AO}{2} = BO \times AO$

$= \frac{a}{\cos\theta} \times \frac{a}{\sin\theta}$

$= \frac{2a^2}{2\sin\theta\cos\theta}$

$= \frac{2a^2}{\sin 2\theta}$ 3.

(b) $\frac{dS}{d\theta} = -2a^2(\sin 2\theta)^{-2} \times 2\cos 2\theta.$

$= \frac{-4a^2\cos 2\theta}{\sin^2 2\theta}$

now ~~increasing~~ decreasing. where $\frac{-4a^2\cos 2\theta}{\sin^2 2\theta} > 0$ for $0 < \theta < \frac{\pi}{2}$.

ie $\cos 2\theta > 0$ for $0 < 2\theta < \pi$.

$0 < 2\theta < \frac{\pi}{2}$ 2

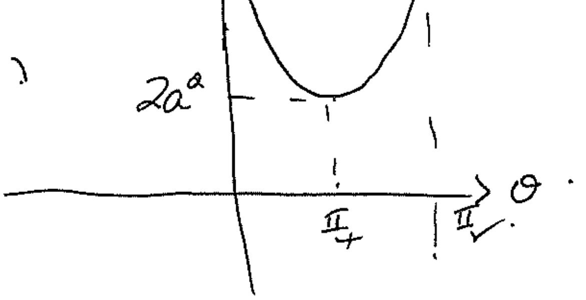
$\boxed{0 < \theta < \frac{\pi}{4}}$

increasing ~~decreasing~~ where $\cos 2\theta < 0$ for $0 < 2\theta < \pi$

$\frac{\pi}{2} < 2\theta < \pi$ 2.

$\boxed{\frac{\pi}{4} < \theta < \frac{\pi}{2}}$

(c)



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(d) $2a < \frac{a}{\sin\theta} < 3a$

$2 < \frac{1}{\sin\theta} < 3$

$\frac{1}{2} > \sin\theta > \frac{1}{3}$

where $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

$\therefore S = \frac{a^2}{\frac{1}{2} \times \frac{\sqrt{3}}{2}} = \frac{4a^2}{\sqrt{3}}.$ 3

where $\sin\theta = \frac{1}{3}$ $\cos\theta = \sqrt{1 - \frac{1}{9}} = \frac{\sqrt{8}}{3}$

$\therefore S = \frac{a^2}{\frac{1}{3} \times \frac{\sqrt{8}}{3}} = \frac{9a^2}{\sqrt{8}}$ $\therefore$ MAX.